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CRITICAL POINTS IN THE THEORY OF ELECTRON BEAM DEVICES

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Abstract—In working on the theory of electron beam devices such as travelling-wave tubes and klystrons, it has been necessary to consider a number of knotty problems, some at least of which occur in mathematical formulation of other plasma problems. Among these is the matter of multi-velocity flow. Here there are two mathematically equivalent approaches. In one, the electron flow is divided into streams according to the initial unperturbed velocities of the electrons; the variables are the densities and velocities in the various streams. In the other, velocity is regarded as a co-ordinate of phase space and the variable is the density in phase space. These approaches give different appreciations of the phenomena involved and lead to different mathematical difficulties, but are the same in content. Each demonstrates the non-existence of wave-type solutions in many infinite velocity distributions. Only a few specialized solutions of multi-velocity problems exist.

As different velocity co-ordinates can be used, so different spatial co-ordinates can be used. The Eulerian approach is common and leads to no difficulties if boundary conditions are met at fluctuating boundaries. In the case of thin beams, it is sometimes useful to use displacements from the mean position of the particles as variables. Some workers have made mistakes by mixing co-ordinate systems.

In dealing with power flow in electron beams, one can either replace the actual physical system with a linear system and find an expression for power in this system, or try to deal with the power flow in the true non-linear system. The former alternative is much simpler. Kinetic power, as well as electromagnetic power, is important in considering the orthogonality of wave-type components.

In solving an actual physical problem, one can either assemble various wave-type components so as to satisfy the boundary conditions, or solve the problem by transform or perhaps by other means.

Components of the solution must be regarded as merely a part of the solution of an actual problem, but they can sometimes be given a reasonable physical interpretation. Thus, one finds waves with positive and negative powers. When two unattenuated waves having powers of the same sign are coupled together, one observes beats. When two unattenuated waves having powers of opposite signs are coupled together, one observes growing and decaying waves. Growing waves can also be produced when a moving discontinuity couples two unattenuated waves together (parametric amplification).

THIS paper is concerned with some aspects of disturbances in clouds of charged particles. This is a particularly exasperating field of physics. What one ordinarily hopes of theoretical work is one of two things. One may hope to be able to work out an exact theory. This can be used either to predict or explain the behaviour of physical systems, or to seek for new physical effects in addition to those encompassed in the theory. Celestial mechanics approaches this ideal and so does the propagation of electromagnetic waves in passive isotropic media. On the other hand, one may be dealing with a physical phenomenon concerning which very accurate measurements can be made but concerning which one cannot work out an exact theory. In this case the validity of an approximate theory can be accurately gauged by comparing it with precise physical measurements.

I think it is no secret that it has been impossible to treat any realistic problem involving clouds of charged particles exactly. At the same time, while measurements of the behaviour of clouds of charged particles exhibit many startling phenomena, over many ranges of parameters such measurements are seldom if even highly precise. In fact, except in the case of slightly ionized plasmas they are seldom precise enough to gauge accurately the validity of various refinements or approximations made in the theory. Thus the theorist is at liberty either to make drastic simplifications or to make refined and perhaps speculative approximations, and it appears that experiments seldom catch up with him.

While measurements on electron beam devices do not have anything like the accuracy of measurements of the spectra of atoms and molecules they have perhaps been a little better than some other plasma measurements. Certainly, the experimental work has determined the need for and value of theory in connexion with such devices. It seems to me to be of some interest to consider what matters have been important in connexion with such devices and some of the things which have been learned by studying them. In dealing with these matters I shall cite recent papers as well as original sources. I do this because a host of papers can be reached by means of the citations in later papers, while this is not true in the case of the original sources.

I think that with a few exceptions which consist of Monte-Carlo attacks on problems of high-level operation and of noise, all electron beam calculations assume a smoothed-out flow of charge rather than a cloud of particles. Thus, collisions between particles are disregarded; the charge interacts only through collective phenomena. It appears that in electron beam devices these collective interactions are much more important than interactions due to the graininess of the flow. However, one effect of this graininess must be taken into account; the noise to which it gives rise. This noise can be introduced into calculations by assigning appropriate fluctuations to the smoothed-out flow in the vicinity of the cathode.

While many useful calculations can be made in which the velocity spread of the electrons is disregarded, both experiment and theory show that this leads to errors in the computation of noise.

There are two ways of handling the problem of multi-velocity smoothed-out electron flow. In each case the equations are linearized. The two methods are equivalent in content, but the equations look somewhat different. I shall illustrate these for a one-dimensional case.

In what we may call the multi-stream approach (PIERCE, 1948) we regard the electron flow as being made up of a number of streams, and we distinguish between the unperturbed velocity u, different for every stream, and the associated a.c. or perturbation electron velocity which we call v_u. In the following equations which pertain to this case I have assumed that each quantity contains a factor

$$\exp(-j\beta z)\exp(j\omega t).$$

The charge density $\rho(u)$ of electrons in the velocity range du at u is given in terms of the total average charge density ρ_0 and a density function $\rho(u)$ by equations (1) and (2):

$$\rho(u) = \rho_0 P(u)\,du, \tag{1}$$

$$\int_{-\infty}^{\infty} P(u)\,du = 1. \tag{2}$$

We may take as the two perturbation or *a–c* variable quantities v_u, the a.c. velocity of electrons with an average velocity u, and i, the total convection current density. These are given in terms of the a.c. field E by (3) and (4):

$$v_u = \frac{j(e/m)E}{(\omega - \beta u)}, \tag{3}$$

$$i = j\omega(e/m)\rho_0 E \int_{-\infty}^{\infty} \frac{P(u)}{(\omega - \beta u)^2}\,du. \tag{4}$$

Here e/m, the charge-to-mass ratio of the electron, is taken as a positive quantity.

In another approach the co-ordinates used are different. The particle velocity w is regarded as a co-ordinate rather than a variable. Thus we have a density $f(t, z, w)$ which is the charge density in phase space (SIEGMANN, 1957). The equation we deal with is Liouville's theorem, which states that the density in phase space in the vicinity of any one particle does not change with time. As we disregard individual particle interactions we deal with a phase space of six dimensions. We can express the total derivative with respect to time in terms of partial derivatives taken at a fixed point, and we can express the rate of change of velocity with time in terms of the electric field. Thus, for the one-dimensional case we obtain

$$\frac{df}{dt} = 0, \tag{5}$$

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial z}\frac{dz}{dt} + \frac{\partial f}{\partial w}\frac{dw}{dt} = 0,$$

$$\frac{\partial f}{\partial t} + w\frac{\partial f}{\partial z} = (e/m)E\frac{\partial f}{\partial w}. \tag{6}$$

These equations can be linearized by representing f as the unperturbed distribution function plus a small perturbation component, and by neglecting this perturbation component on the right-hand side of (6).

In the course of our calculations we will need various quantities such as the total charge density and the average velocity, both of which have a.c. components. These two quantities are listed in (7) and (8):

$$\rho = \int_{-\infty}^{\infty} f\,dw, \tag{7}$$

$$v = \frac{\int_{-\infty}^{\infty} wf\,dw}{\rho}. \tag{8}$$

The important application of multi-velocity flow in vacuum tubes has been, as I have noted, in the computation of noise. Here it seems necessary to resort to numerical solutions of the equations. One divides the electron flow into velocity classes, assumes a sinusoidal excitation in one class at the cathode and computes the over all excitation at various points along the flow. This procedure is repeated with a sinusoidal excitation in each class in turn. A mean-square excitation appropriate to uncorrelated excitations in the various streams is obtained by adding the squares of the amplitudes due to the various initial sinusoidal excitations. This has been done by SIEGMAN, WATKINS and HSIEH (1957), by PIERCE and MORRISON (1958) and by CURRIE and MUELLER (to be published). In such calculations it has been found best to use the density function rather than the multistream formulation of the problem because fewer equations have to be integrated.

CURRIE and MUELLER'S calculations show, for example, that the noise in electron flow accelerated by a given voltage decreases as the accelerating field is made smaller and longer; this effect is absent in single-velocity approximations. A noise lower than that predicted by single-velocity calculations is observed experimentally, and this noise is in at least rough numerical agreement with the multi-velocity computations. The multi-velocity calculations have not been carried out for the complicated geometries actually used, but for a parallel plane geometry with a specified d.c. potential.

In a particular noise problem TIEN and MOSHMAN (1956) proceeded not from the equations for smoothed-out flow, but from the dynamics of individual particles, and calculated noise by a Monte-Carlo method.

Such numerical computations are of great interest to tube designers. In many cases, however, what we would like is a little general insight into the effect of velocity distributions. To this end we can look for one-dimensional wave-like solutions in a disturbed plasma. Suppose, for instance, that we use (4) together with the law that the total a.c. current must be 0.

$$i + j\omega E = 0. \tag{9}$$

This problem was investigated by LANDAU (1946) for a Maxwellian velocity distribution and by PIERCE and MORRISON (1959) for a velocity distribution in which the charge density function is the reciprocal of a constant plus the velocity squared.

For both of these cases one indeed finds a disturbance with a decaying sinusoidal oscillating electric field. However, various components of charge density have fluctuations of other frequencies; it is only the overall charge density that has a simple exponential variation with time and distance. Thus, the disturbance is not what we usually consider to be a wave, if we say that in a wave all perturbation quantities vary in the same way with time and distance. Further, for such multi-velocity flow it can be shown that the linearized solution becomes invalid at large times even for small initial disturbances (PIERCE and MORRISON, 1959).

We can seek further insight in various ways. For instance, we can consider the behaviour of a flow consisting of a number of streams of charge densities ρ_n and velocities u_n. In this case we are faced with equation (10):

$$\frac{(e/m)}{\varepsilon}\sum_n \frac{\rho_n}{(\omega - \beta u_n)^2} = 1. \tag{10}$$

In Fig. 1, the left-hand and right-hand sides of equation (10) have been plotted as functions of the phase constant β for a case in which streams of three velocities are present. The phase constants of the waves which make up the solution correspond to the intersections of the curve for the left-hand side of the equation with the horizontal line which represents the right-hand side of the equation. The solutions consist of an unattenuated wave for which β is less than ω/u_1, where u_1 is the highest velocity, an unattenuated wave for which β is greater than ω/u_3, where u_3 is the lowest velocity, and two pairs of waves, each embracing a growing and a decaying wave. The rate of growth becomes very small when the velocities lie very close together. The six waves allow us to satisfy velocity and convection current boundary conditions for the three streams. The two waves whose values of β lie outside of the range of ω/u can be thought of as waves involving the whole electron flow. The other waves tend to be confined largely to particular velocity ranges of electrons, and thus the overall solution consists of two major waves plus some disturbances which we might regard as being convected with the various components of the electron flow. We should notice, however, that all of these disturbances are necessary to solve a physical problem with particular boundary conditions, such as a noise problem, and that the two unattenuated waves are not sufficient for such a purpose.

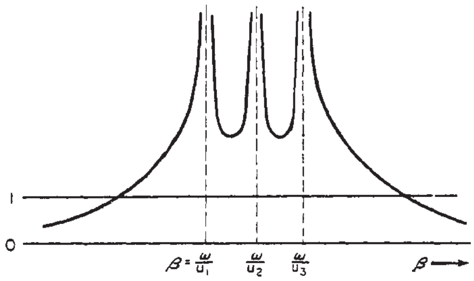

FIG. 1.—The left-hand and right-hand sides of an equation for waves in one-dimensional, three-stream flow.

Rather than approximating the multi-velocity flow by a number of discrete electron streams, we can expand the integrand of equation (4) in powers of $\beta u/\omega$, as in (11):

$$\frac{1}{(\omega-\beta u)^2}=\frac{1}{\omega^2(1-\beta u/\omega)^2}$$
$$=\frac{1}{\omega}(1+2(\beta u/\omega)+3(\beta u/\omega)^2+\ldots) \tag{11}$$

Thus the integral becomes

$$\int_{-\infty}^{\infty}\frac{P(u)}{(\omega-\beta u)}\,du=\frac{1}{\omega^2}(1+2(\beta/\omega)\langle u\rangle + 3(\beta/\omega)^2\langle u^2\rangle+\ldots). \tag{12}$$

If we assume the average velocity to be zero and disregard moments higher than the second, we obtain a relation which is commonly known as the dispersion relation of BOHM and GROSS (1949)

$$\omega^2=\omega_p{}^2(1+3(\beta/\omega)^2\langle u^2\rangle). \tag{13}$$

We should note several things about this dispersion relation. If the velocity distribution is infinite, the series (11) diverges, and this is true for a Maxwellian distribution. Thus, the dispersion relation cannot be rigorously derived from the original equation. For some infinite velocity distributions it indeed does not give the same frequency of oscillation as the exact solution for the smoothed-out flow does (PIERCE and MORRISON, 1959). For the Maxwellian distribution, however, for small values of $(\beta/\omega)^2\langle u^2\rangle$, that is, for long wavelengths, the frequency is correct and the decay of the exact solution is very small. Thus the dispersion relation has some validity for the Maxwellian velocity distribution when $(\beta/\omega)^2\langle u^2\rangle$ is sufficiently small. It is dangerous, however, to apply it carelessly to large values of $(\beta/\omega)^2\langle u^2\rangle$, and it does not in any case completely describe disturbances in the plasma.

We may note that while we must make approximations which in some cases are dubious in dealing with multi-stream flow according to equations (4) and (9), these equations themselves are approximate in that they disregard the particular nature of the electron flow. Thus, seemingly important mathematical features can be concerned with velocity ranges in which the probability of finding a particle is essentially zero. What frame of mind this should leave one in I do not know. An optimist can argue that any approximation in dealing with the equations is justified in that the equations themselves are only approximate. I wish we had comparisons between predicted behaviour and actual behaviour accurate enough to test all those features of approximate solutions which are alleged to have practical importance. This seems to me to be particularly desirable when we go beyond equations in which collisions are disregarded and try to take the effect of collisions into account.

It is difficult to treat even the simplest problems of multi-velocity flow in the case of finite beams (PIERCE and WALKER, 1953 and 1956). Moreover, most of the behaviour of electron beam devices can be satisfactorily dealt with by disregarding not only the discrete nature of the electron flow but thermal velocities as well and the rest of this paper will be concerned with single-velocity flow. We have noted that in considering multi-velocity flow we have some choice as to co-ordinates, and indeed there is a wide choice of co-ordinates in dealing with single-velocity flow. These have been discussed by BOBROFF (1959). It is worth mentioning the matter because occasionally workers have mixed up co-ordinate systems and got wrong answers.

The two most common sets of co-ordinates are illustrated in the 2-dimensional case in Fig. 2. Above, we have Eulerian co-ordinates appropriate to a thick beam. These consist of values of various variables at a particular point in the flow, and the variables taken are ρ, the charge density, v_x and v_y, the velocity components of particles at a point and E_x and E_y, the a.c. electric field components. To these we may add $\tilde{y}$, the displacement of the beam boundary. This displacement $\tilde{y}$ is important because boundary conditions must be met at the wavy boundary between the beam and the charge-free space outside. However, this can be managed by assuming a surface current or surface charge proportional to $\tilde{y}$ (HAHN, 1939; RAMO, 1939; RIGROD and PIERCE, 1959).

Sometimes it is convenient to make calculations concerning a thin beam of electrons, as illustrated in the lower part of Fig. 2. Here appropriate variables are the surface charge density σ, v_x, v_y, E_x, E_y and $\tilde{y}$

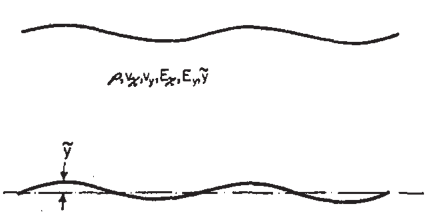

FIG. 2.—Co-ordinates for a two-dimensional thick beam and a two-dimensional thin beam.

(Pierce, 1956). It is unnecessary to use both v_y and $\tilde{y}$, and, indeed, $\tilde{x}$ and $\tilde{y}$ can be used in place of v_x and v_y.

Power flow is a matter of primary importance in connection with disturbances on electron beams. This has been treated in a general manner by Haus and Bobroff (1957). Here I shall illustrate the matter of power flow in the case in which the electrons are constrained to move in the z direction only, as by an infinite magnetic field (Louisell and Pierce, 1955).

Suppose we start with the general non-linear equations governing the electron stream, the force equation (14) and the definition of current density given by (15):

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{e}{m}E, \tag{14}$$

$$J = \rho v. \tag{15}$$

Using these and Maxwell's equations, we seek quantities which will satisfy equation (16):

$$\frac{\partial}{\partial z}(P_e + P_k) + \frac{\partial}{\partial t}(W_e + W_k) = 0. \tag{16}$$

Here P_e and W_e are electromagnetic power flow and stored energy density and P_k and W_k are kinetic power flow and energy density. It is clear that these quantities must be given by equations (17) through (20):

$$P_e = (\mathbf{E} \times \mathbf{H})_z, \tag{17}$$

$$W_e = \tfrac{1}{2}\,\mu\mathbf{H}\,.\,\mathbf{H} + \tfrac{1}{2}\,\varepsilon\mathbf{E}\,.\,\mathbf{E}, \tag{18}$$

$$P_k = -\frac{1}{2}\left(\frac{m}{e}\right)Jv^2. \tag{19}$$

$$W_k = -\frac{1}{2}\left(\frac{m}{e}\right)\rho v^2 \tag{20}$$

We note that *all* electric magnetic energies associated with the flow appear in W_e.

Almost always in vacuum tube work we deal with linearized equations. Certainly we cannot take the first-order approximations obtained through the solutions of linearized equations and safely use them in the exact expressions for a.c. power, which contain second order terms. What are we to do?

Two courses are open to us. One is to carry along higher-order terms in our solutions of the true non-linear equations. Walker has done this difficult task (Walker. 1954, 1955). Another simpler and satisfactory approach is to linearize the problem, not the solution (Louisell and Pierce, 1955). Thus, we regard the linearized force and current equations.

$$\frac{\partial v}{\partial t} + u_0\frac{\partial v}{\partial z} = -\frac{e}{m}E_z, \tag{21}$$

$$\left.\begin{aligned} J &= \rho u_0 + \rho_0 v, \\ J_0 &= \rho_0 u_0, \end{aligned}\right\} \tag{22}$$

as the equations of a system which we are considering, a system whose behaviour will correspond closely to the behaviour of the actual non-linear system at low signal levels. We then seek to find for this linear system definitions of P_k and W_k which will satisfy (16), Maxwell's equations and equations (21) and (22). A little diligent search leads us to expressions (23) and (24) for P_k and W_k:

$$P_k = -\frac{1}{2}\left(\frac{m}{e}\right)(J_0 + J)(u_0^2 + 2u_0 v), \tag{23}$$

$$W_k = \frac{1}{2}\left(\frac{m}{e}\right)[\rho_0(u_0^2 + 2u_0 v + v^2) + \rho(u_0^2 + 2u_0 v)]. \tag{24}$$

We should note that these do not contain all possible second-order terms.

For sinusoidal excitations the a. c. power density may be written simply in terms of the convection current density J and a kinetic voltage U, as indicated in (25) and (26):

$$P_k = \tfrac{1}{2}(JU^* + J^*U), \tag{25}$$

$$U = \frac{-u_0 v}{(e/m)}. \tag{26}$$

The presence of kinetic power as well as of electromagnetic power is important in connexion with the orthogonality of modes of propagation in an electron stream. For purely electromagnetic waves the condition of orthogonality is that the power in the direction of propagation computed by using the electric field of one mode and the magnetic field of another mode is zero. However, orthogonality must involve total power, electromagnet and kicinetic. Thus the orthogonality relation is

$$0 = \int[(\mathbf{E}_n \times \mathbf{H}_m^* + \mathbf{E}_n^* \times \mathbf{H}_m) + (\mathbf{U}_n\,.\,\mathbf{J}_m^* + \mathbf{U}_n^*\,.\,\mathbf{J}_m)]\,ds, \qquad m \neq n \tag{27}$$

Here the field components are normal to the direction of propagation and **J** and **U** are along the direction of propagation; the integral is over the area of the wave. Bressler, Joshi and Marcuvitz (1958) have dealt with this matter.

Ordinarily it is most convenient to think of the excitation of an electron stream in terms of modes of propagation, but these are after all only the components of a solution to an actual physical problem. One can solve the problem either by finding all the relevant modes and fitting the boundary conditions, or he can use some sort of transform method (Rigrod and Pierce, 1959; Walker, 1954). Either in the search for the modes or in the solution of problems by transform methods, admittance and impedance can be very useful concepts. Consider, for example, an axially symmetrical disturbance on a cylindrical electron stream with some specified phase constant and frequency. At some point outside the stream there will be a certain circumferential magnetic field and an axial electric field. At any radius we can regard the ratio of H_θ to E_z as an admittance. In some cases there may be another component of admittance involving E_θ and H_z. H_θ also gives the total current I_z within the radius R. Thus the admittance Y is

$$Y = \frac{H_\theta}{E_z} = \frac{I_z}{E_z}. \tag{28}$$

Once we have the admittance Y, we can consider the problem of the electron stream surrounded by a conducting tube at the radius R. We can, for instance, specify E_z as zero except across an infinitesimal gap at $z = 0$ and thus find the current everywhere in the wall of the tube resulting from excitation of the beam by a voltage across a gap in the tube (RIGROD and PIERCE, 1959).

We can also use Y in finding the normal modes. The rule is that for a normal mode the sum of the admittances looking in toward the beam and out away from the beam must be zero,

$$Y_{\text{in}} + Y_{\text{out}} = 0 \tag{29}$$

The admittance looking outward into a conducting tube is infinite and thus the normal modes for a beam inside a conducting tube at radius R occur at the poles of the admittance looking inward.

The admittance Y looking in toward a thick electron beam usually has a large number of poles for values of ω and β such that the frequency ω_m seen moving with the electrons is nearly equal to the frequency ω_p.

$$\omega_m = |\omega - \beta u_0|, \tag{30}$$

$$\omega_p = \sqrt{\frac{(e/m)\rho_0}{\varepsilon}}. \tag{31}$$

There are also a large number of poles when ω_m is near to the cyclotron frequency ω_c,

$$\omega_c = (e/m)B. \tag{32}$$

The different roots in these vicinities correspond to different variations of field quantities with radius. These modes have been studied in considerable detail by HAHN (1939) and RAMO (1939).

Let us for the moment disregard the effects of the cyclotron frequency and consider the two space-charge waves which correspond to the simplest and least rapid field variation with radius. One of these will be a wave whose phase velocity is faster than the electron velocity and the other will be a wave whose phase velocity is slower than the electron velocity. The fast wave has positive energy and positive power. The slow wave has a negative energy and a negative power.

How does this come to be? We can understand by considering the behaviour of the electrons in a coordinate system which moves with the phase velocity of the wave. In this case we will, in effect, see the electrons moving against and with the electric field much as balls might roll over a sinusoidal hill, as shown in Fig. 3. When the phase velocity of the wave is less than the velocity of the electrons the balls will be rolling to the right, in the direction of wave motion. They will be bunched together at the tops of the hills, where their velocity with respect to the wave is least. The velocity with respect to a fixed frame of reference is the velocity of the wave plus the velocity of the electrons with respect to the wave. Thus, the electrons are bunched together in positions at which their total velocity with respect to the fixed frame of reference is least. This means that in order to set up such a disturbance, energy must be abstracted from the wave, and so we must assign a negative kinetic energy and a negative kinetic power to the wave.

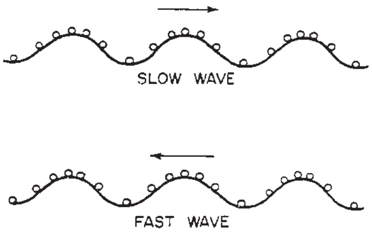

FIG. 3.—Representation of motion of electrons in the field of a slow (above) and a fast (below) wave, as seen moving with the phase velocity of the wave.

As shown also in Fig. 3, when we move with the fast wave we see the electrons as moving backward, to the left. Again they are bunched in regions of least velocity with respect to the wave. In this case, however, the velocity of the electrons in the fixed frame of reference is the velocity of the wave minus the velocity of the electrons with respect to the wave. Thus, the velocity in the fixed frame is greatest where the velocity of the electrons with respect to the wave is least, that is, where the electrons are bunched together. For this reason the fast wave has positive kinetic energy and positive kinetic power.

Both the slow and the fast waves have some electromagnetic power as well as their kinetic power, but for space-charge waves whose velocity is slow compared with the velocity of light the kinetic power predominates.

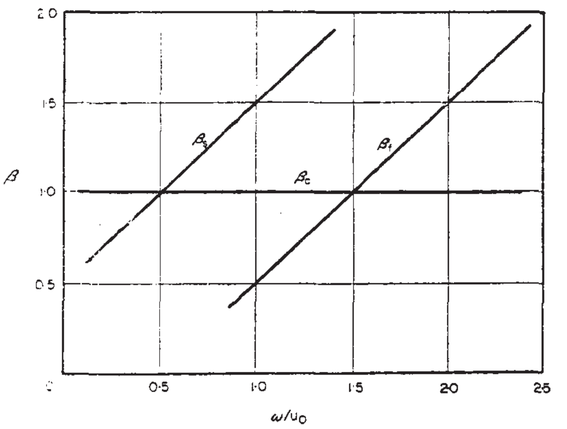

FIG. 4.—Phase constants of circuit and space-charge waves plotted against a parameter involving electron velocity u_0.

The performance of many types of electron tubes can be explained in terms of the energy of space-charge waves. Interesting phenomena in tubes can be explained in terms of the coupling of modes of propagation (PIERCE, 1954). The effect of such coupling depends on whether the signs of the powers of the waves which are coupled are the same or are different.

To illustrate this, let us consider the case of an electron beam coupled to a helix, as in the travelling-wave tube. We will seek the behaviour of the modes as we vary the velocity u_0 of the electrons. In Fig. 4 the phase constant β for various waves is plotted vertically: horizontally there is plotted a parameter ω/u_0, which changes as the electron velocity u_0 is changed. β_c is the phase constant of waves on the circuit which, of course, does not change as the electron velocity is changed. β_s is the phase constant of the slow space-charge wave on the electron stream, and β_s is the phase constant of the fast space-charge wave on the electron stream. We may expect some interaction for values of u_0 which make β_s or β_f close to β_c.

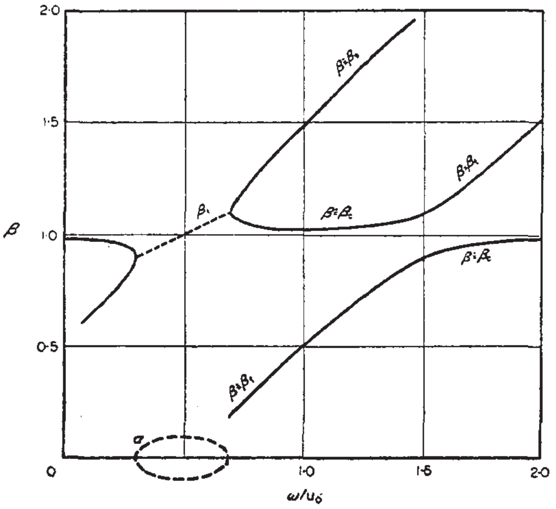

FIG. 5.—Result of coupling modes of Fig. 4.

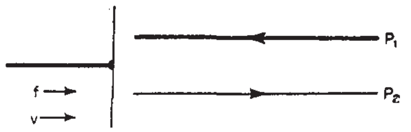

FIG. 6.—Reflection of a wave of power P_1 from an advancing piston.

The interaction results in the variation of phase constant with electron velocity shown in Fig. 5. When the powers are different in sign, as in the case of the circuit wave and the slow space-charge wave, the coupling results in a pair of waves with the same phase constant, one increasing exponentially with distance and the other decaying exponentially with distance. Each is made up of a component of circuit wave and a component of slow space charge carrying equal and opposite powers, so that the total power of the decreasing wave is zero and the total power of the increasing wave is zero. The gain of the increasing wave represents, not a gain in net power, but a gain in the power of the positive-power component wave at the expense of an increasing negative power in the negative-power components.

When the powers of the two waves are both negative or both positive, as in the case of the circuit wave and the fast space-charge wave, the coupling merely results in two unattenuated waves, as shown in Fig. 5.

One other mechanism of interaction has become important in electron tubes. Consider the case of a wave of power P_1 reflected by a piston which advances with a velocity v and exerts a force f, as shown in Fig. 6. It is clear that the reflected power P_2 must be greater than P_1 and must consist of a wave of a higher frequency. Reflection from a single piston is not a stationary process, but reflection from a multitude of discontinuities, such as those caused by bunches in an electron stream, can be a continuous process, and such *parametric amplification* can be attained in various forms of vacuum tubes as well as in other electromagnetic devices (PIERCE, 1959).

REFERENCES

BOBROFF D. L. (1959) *IRE Trans.* **ED-6**, 1, 68.
BOHM D. and GROSS E. P. (1949) *Phys. Rev.* **75**, 1851.
BRESSLER A. D., JOSHI G. H. and MARCUVITZ N. (1958) *J. Appl. Phys.* **29**, 794.
CURRIE M. R. and MUELLER W. M. *J. Appl. Phys.* To be published.
HAHN W. C. (1939) *Gen. Elect. Rev.* **42**, 258.
HAUS H. A. and BOBROFF D. L. (1957) *J. Appl. Phys.* **28**, 694.
LANDAU L. (1946) *J. Phys., U.S.S.R.* **10**, 25.

LOUISELL W. H. and PIERCE J. R. (1955) *Proc. Inst. Radio Engrs. N.Y.* **43,** 425.
PIERCE J. R. (1948) *J. Appl. Phys.* **19,** 231.
PIERCE J. R. (1954) *Bell. Syst. Tech. J.* **33,** 1343.
PIERCE J. R. (1956) *IRE Trans.* **ED-3,** 183.
PIERCE J. R. (1958) *Proc. Inst. Elect. Engrs.* **105,** Part B Suppl. No. 11, 786.
PIERCE J. R. (1959) *J. Appl. Phys.*
PIERCE J. R. and MORRISON J. A. (1959) *IRE Trans.* **ED-6,** 2, 231.
PIERCE J. R. and WALKER L. R. (1953) *J. Appl. Phys.* **24,** 1328.
PIERCE J. R. and WALKER L. R. (1956) *Bell. Syst. Tech. J.* **35,** 109.
RAMO S. (1939) *Phys. Rev.* **56,** 276.
RIGROD W. W. and PIERCE J. R. (1959) *Bell Syst. Tech. J.* **38,** 99.
SIEGMAN A. E. (1957) *J. Appl. Phys.* **28,** 1132.
SIEGMAN A. E., WATKINS D. A. and HSUNG-CHENG HSIEH (1957) *J. Appl. Phys.* **28,** 1138.
TIEN P. K. and MOSHMAN J. (1956) *J. Appl. Phys.* **27,** 1067.
WALKER L. R. (1954) *J. Appl. Phys.* **25,** 615; (1955) *J. Appl. Phys.* **26,** 1031.